Summer Assignment Calculus AB/Dual Credit   2022

Algebra/Precalculus skills are essential to calculus courses. This packet includes skills that are required to be successful in calculus. You will receive a homework grade based on completion of the problems on the first day of school. We will have quizzes on Wednesdays ("What Do You Know Wednesdays") that will include these skills in addition to new skills learned in the course.

Please feel free to reach out ot me via email over the summer if you have any questions.

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Finding the Slope of a Line Algebraically:

How do we find the slope (denoted as m) of a line given two points (x_1, y_1) and (x_2, y_2)?

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example: Find the slope of the line containing the points $(-6, -1)$ and $(3, 2)$.

$$m = \frac{2 - (-1)}{3 - (-6)} = \frac{3}{9} = \frac{1}{3}$$

Find the slope for the line that contains each pair of points.

a) $(2, 3)$ and $(4, 15)$

b) $(3, 6)$ and $(5, 4)$

c) $(-2, -5)$ and $(3, 7)$

d) $(-\frac{1}{2}, 0)$ and $(2, \frac{3}{2})$

e) $(2, -4)$ and $(-1, 5)$

f) $(\frac{1}{4}, 2)$ and $(\frac{3}{4}, \frac{5}{2})$

Writing an Equation of a Line Using Point-Slope Form:

What is point-slope form?

$y - y_1 = m(x - x_1)$

Example: Find the equation of the line containing the points $(-6, -1)$ and $(3, 2)$ using point-slope form.

Solution: First find the slope between the two points.

$$m = \frac{2 - (-1)}{3 - (-6)} = \frac{3}{9} = \frac{1}{3}$$

Now, write the equation of the line.

$$y - (-6) = \frac{1}{3}(x - (-1))$$

$$y + 6 = \frac{1}{3}(x + 1)$$

Write an equation for each line in point-slope form.

a) containing the point $(4, 2)$ and with a slope of $\frac{1}{2}$

b) containing the point $(-1, 3)$ and with $m = 2$

c) containing the points $(1, 1)$ and $(4, 5)$.

Factoring Polynomials:

Below is the thought process for factoring polynomials!

Step 1: Identify the GCF between the terms and factor it out if the GCF $\neq 1$. If the leading term (term with the highest power) is negative then you must also factor out a negative first! Then look inside the parentheses and see if it can factored further.

Step 2: If the GCF between all the terms is 1, then look to see how many terms you have!

Technique for 4 terms:

Use group factoring! Technique is explained below.

- Group the first two terms and the last two terms together.
- Then factor out the GCF from each of the two sets of parentheses.
- After you factor out the GCF's, looking at your new expression with two terms, factor out the GCF again which will be a binomial.

Techniques for 3 terms (trinomials): For trinomials of the form $\text{a}x^2 + bx + c$

a = 1: (technique explained below)

Think of factors of the c-value that add up to b-value

$a \neq 1$: (technique explained below)

- Multiply the a and c value and find factors that somehow add/subtract to the middle term.
- Expand out the expression into 4 terms.
- Use group factoring method to factor the expression.

Techniques for 2 terms (binomials):

Difference of perfect squares

$a^2 - b^2 = (a - b)(a + b)$

Sum/Difference of perfect cubes

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

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Factor the polynomials below completely!

a) $2x^2 + 12x + 16$

b) $x^2 - 3x + 10$

c) $x^2 + 2x - 24$

d) $x^2 - 16$

e) $8x^3 + 27$

f) $2x^3 - 18x$

g) $x^3 - 4x^2 + 2x - 8$

h) $5x^3 - x^2 - 5x + 1$

i) $5x^2 - 3x - 2$

j) $3x^2 + 7x + 2$

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Zero Product Property: This property states that if $ab = 0$ then either $a = 0$ or $b = 0$ (or both are zero).

Using the Zero Product Property to solve equations:

Example: Solve $(3x + 1)(x - 2) = 0$ for x.

$3x + 1 = 0$ or $x - 2 = 0$

$3x = -1$ or $x = 2$

$x = -\frac{1}{3}$ or $x = 2$

Solve the equations below for x.

a) $2x(x - 1) = 0$

b) $(2x + 5)(x - 4) = 0$

c) $x(3x + 4)(2x - 1) = 0$

d) $(x - 3)(x + \frac{1}{2}) = 0$

e) $(\frac{3}{2}x + 4)(2x - \frac{1}{3}) = 0$

f) $2(x - 10)(4x + 5) = 0$

Solving Quadratic Equations by Factoring:

Example 1: Solve $15x^2 + 5x = 0$ for x.

Factor the left hand side of the equation and use the zero product property to solve for x.

$5x(3x + 1) = 0$

$5x = 0$ or $3x + 1 = 0$

$x = 0$ or $x = -\frac{1}{3}$

Example 2: Solve $x^2 + 3x - 14 = -2x + 10$ for x.

Bring all the terms to one side of the equation. Then, factor the left hand side of the equation and use the zero product property to solve for x.

$x^2 + 5x - 24 = 0$

$(x + 8)(x - 3) = 0$

$x + 8 = 0$ or $x - 3 = 0$

$x = -8$ or $x = 3$

Solve the equations below for x by factoring.

a) $12x^2 + 8x = 0$

b) $x^2 + 5x + 4 = 0$

c) $3x^2 + 11x - 6 = -2$

d) $4x^2 + 3x - 11 = 3x - 2$

Exponent Rules:

- $x^m \cdot x^n = x^{m+n}$ Ex: $2x^2 \cdot 3x^5 = 6x^{2+5} = 6x^7$
- $\dfrac{x^m}{x^n} = x^{m-n}$ Ex: $\dfrac{8x^9}{2x^5} = 4x^{9-5} = 4x^4$
- $(x^m)^n = x^{mn}$ Ex: $(2x^3)^2 = 2^2x^{3\cdot 2} = 4x^6$
- $x^0 = 1$ where $x \neq 0$ Ex: $(9xyz)^0 = 1$
- $x^{-n} = \frac{1}{x^n}$ Ex: $x^{-3} = \dfrac{1}{x^3}$
- $\sqrt[n]{x^m} = x^{\frac{m}{n}}$ Ex: $\sqrt[4]{x^3} = x^{\frac{3}{4}}$

Remember that for the square root symbol the root number is 2 but it is never labeled.

Simplify the expressions and make sure to use positive exponents in your final answer.

a) $7x^6 \cdot 2x^4$

b) $\dfrac{6x^7y^8}{2x^3y}$

c) $-2(8xy)^0$

d) $(4x^4y^{-2})^3$

e) $(3ab^6)^2(-2a^2b^{-2})^3$

f) $\sqrt{\dfrac{x^2}{y^2}}$

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Rewrite the following expressions using rational exponents.

Example: $\frac{1}{\sqrt[3]{x^2}} = \frac{1}{x^{\frac{2}{3}}} = x^{-\frac{2}{3}}$

a) $\sqrt[5]{x^3} + \sqrt{x}$

b) $\sqrt[3]{x+1}$

c) $\frac{1}{\sqrt[3]{x^5}}$

d) $\frac{1}{x^{10}} - \frac{3}{x}$

e) $\frac{1}{2x^6} + \frac{1}{4\sqrt{x}}$

f) $2\sqrt{x^7+4}$

Rewrite the following expressions using roots/positive exponents.

Example: $x^{-\frac{2}{3}} + 2x^{-7} = \frac{1}{x^{\frac{2}{3}}} + \frac{2}{x^7} = \frac{1}{\sqrt[3]{x^2}} + \frac{2}{x^7}$

a) $x^{-\frac{5}{2}} + x^{\frac{1}{4}}$

b) $x^{-8} + 6x^{-3}$

c) $(2x+1)^{-16}$

d) $\frac{3}{2}x^{-1}$

e) $(3x^5+10)^{-\frac{1}{2}}$

f) $\frac{1}{(3x)^{\frac{4}{5}}}$

Multiplying Binomials using FOIL Method:

FOIL stands for First Outer Inner Last!

Example: Multiply $(2x + 1)(x + 4)$

Solution: $(2x + 1)(x + 4) = 2x^2 + 8x + x + 4 = 2x^2 + 9x + 4$

Multiply the binomials below and please show work!

a) $(x + 2)(x - 3)$

b) $(3x - 2)(2x + 7)$

Special Formulas for Binomials:

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b)(a - b) = a^2 - b^2$

Expand out the expressions and simplify using the special formulas above for binomials!

Example: $(\sqrt{x} - \sqrt{y})(\sqrt{x} + \sqrt{y}) = (\sqrt{x})^2 - (\sqrt{y})^2 = x - y$

a) $(\sqrt{3a} - \sqrt{4b})(\sqrt{3a} + \sqrt{4b})$

b) $(x^2 + y^2)(x^2 - y^2)$

c) $(2x + 3y)^2$

d) $(\sqrt{x} - 2)^2$

Function Evaluation:

Example: Suppose $f(x) = 2x^2$, find $f(x+1)$.

Solution: $f(x+1) = 2(x+1)^2 = 2(x^2+2x+1) = 2x^2+4x+4$

Suppose $f(x) = x^2 + 5$. Compute the following and please show work!

a) $f(-2)$

b) $f(x^3)$

c) $f(x-3)$

d) $f(2x+1)$

e) $f(x+h)$

f) $\dfrac{f(5)-f(1)}{5-1}$

g) $\dfrac{f(x+h)-f(x)}{h}$

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Function Composition:

Example: Let $f(x) = 2^x$ and $g(x) = 3x + 1$.

$g(f(2)) = g(4) = 3(4) + 1 = 13$ since $f(2) = 2^2 = 4$

$f(g(x)) = f(3x + 1) = 2^{3x+1}$

Let $f(x) = -3x^2 + 1$ and $g(x) = x + 5$. Compute the following!

a) $f(g(0))$ b) $g(f(1))$ c) $g(f(x))$

Let $f(x) = \sqrt{x}$ and $h(x) = 5x - 2$. Compute the following!

a) $h(f(4))$ b) $f(h(3))$ c) $f(h(x))$

Important Properties:

$e^{\ln(x)} = x$ and $\ln(e^x) = x$

Also note that $\ln(e) = \ln(e^1) = 1$ and $\ln(1) = \ln(e^0) = 0$

Simplify the following expressions using the important properties above.

a) $e^{\ln(5)}$ b) $\ln(e^8)$ c) $2e^{\ln(3)} + \frac{1}{3}\ln(e)$

d) $-4\ln(e^{2x}) + \ln(1)$

Solving Exponential Equations:

Example 1: Solve $7^{2x+1} = 7^5$.

Solution: Make sure the exponential's have the same base. Then set the exponents equal and solve for x.

$2x + 1 = 5 \Rightarrow 2x = 4 \Rightarrow x = 2$

Example 2: Solve $e^x = 2$.

Solution: Make sure the exponential is isolated. Then take the natural log of both sides and solve for x.

$e^x = 2 \Rightarrow \ln(e^x) = \ln(2) \Rightarrow x = \ln(2)$

Solve the equations below for x and show your work.

a) $12^{x+5} = 144$

b) $4^{3x-5} = 4^{x+7}$

c) $5^{2x-1} = \sqrt{5}$

d) $e^x + 1 = 4$

e) $3e^x + 5 = 8$

f) $3e^{2x} = 9$

g) $4^x + 5 = 8$

h) $\dfrac{e^x + 1}{2} = 7$

i) $2^{3x+1} = 4^x$

Solving Logarithmic Equations:

How do logarithmic functions work?

For $x > 0$, $b > 0$, and $b \neq 1$, we have that

$$\log_b(x) = y \Rightarrow b^y = x$$

Make sure you isolate the logarithm is the equation first before using the exponential form above!

Example: Solve the equation $\log_2(4x - 12) - 1 = 2$ for x.

Solution: First isolate the logarithm by adding 1 to both sides in this case. Then use its equivalent exponential form to solve for x.

$\log_2(4x - 12) = 3$

$2^3 = 4x - 12 \Rightarrow 8 = 4x - 12 \Rightarrow 4x = 20 \Rightarrow x = 5$

Solve the equations below for x and show your work! Recall that $\ln(x) = \log_e(x)$ and $\log(x) = \log_{10}(x)$

a) $\log_2(x) = 4$

b) $\log_3(x) = -3$

c) $\log_{64}(x) = \frac{1}{2}$

d) $\log_6(2x - 4) = 2$

e) $3\log_2(x + 4) - 5 = 10$

f) $2\ln(x) + 3 = 4$

g) $3\log(3x - 2) = 3$

Finding the Domain of Functions Algebraically:

- **Polynomials, Exponential, Sine, Cosine, and Odd Root Functions:** The domain is all real numbers!

  Set Notation: $\mathbb{R}$ Interval Notation: $(-\infty, \infty)$

  Note: An odd root function is a root function with an odd root number. For example, $f(x) = \sqrt[3]{x}$ is an odd root function.

- **Rational Functions:** Set the denominator equal to zero and solve for x to find x-values not included in the domain. (Remember we cannot divide by zero!)

- **Even Root Functions:** Set the expression under the even root greater than or equal to zero and solve the inequality. Your answer will give you the domain but rewrite it in interval notation. (Remember we cannot take the square root of a negative number. More generally, we cannot take the even root of a negative number.)

  Note: An even root function is a root function with an even root number. For example, $f(x) = \sqrt[4]{x}$ is an even root function.

- **Logarithmic Functions:** Set the expression inside the logarithm strictly greater than zero and solve the inequality. Your answer will give you the domain but rewrite it in interval notation. (Remember we cannot plug in a negative number into a logarithm since it is undefined.)

Find the domain of the functions below and write your answer in interval notation or set notation.

a) $f(x) = x^2 + 2x + 9$ b) $g(x) = \sin(x)$ c) $h(x) = \sqrt[3]{x+5}$

d) $f(x) = 14x^7 + 9x^3 + 100$ e) $g(x) = -3\cos(2x)$ f) $f(t) = t^{100}$

g) $f(x) = \sqrt{2x+7}$

h) $g(x) = \log(2x-10)$

i) $h(x) = \dfrac{x+4}{x^2-9}$

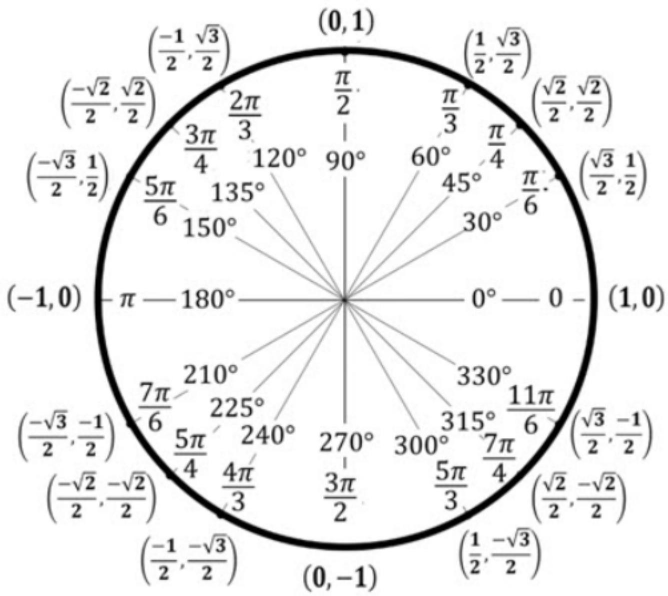
j) $v(t) = -2\ln(x) + 1$

k) $f(x) = \dfrac{1}{x^2+1}$

l) $g(x) = \frac{1}{2}\sqrt[4]{\frac{1}{4}x - 2}$

m) $f(x) = \dfrac{x^2+2x+1}{\sqrt{x+5}}$

n) $g(x) = \dfrac{2x}{x^2+5x+6}$



Use the unit circle below to fill in the tables below! Recall that the coordinates on the unit circle are $(\cos(\theta), \sin(\theta))$.

θ	$\cos(\theta)$	$\sin(\theta)$
0 & 2π		
$\frac{\pi}{6}$		
$\frac{\pi}{4}$		
$\frac{\pi}{3}$		
$\frac{\pi}{2}$		
π		
$\frac{3\pi}{2}$		

θ	$\cos(\theta)$	$\sin(\theta)$
$\frac{5\pi}{6}$		
$\frac{3\pi}{4}$		
$\frac{2\pi}{3}$		
$\frac{7\pi}{6}$		
$\frac{5\pi}{4}$		
$\frac{4\pi}{3}$		

Trigonometric Functions:

Recall the six trigonometric functions: **sine, cosine, tangent, secant, cosecant, cotangent**.

Below are some definitions for the trigonometric functions!

$\tan x = \dfrac{\sin x}{\cos x}$ $\sec x = \dfrac{1}{\cos x}$ $\csc x = \dfrac{1}{\sin x}$

$\cot x = \dfrac{1}{\tan x}$ $\cot x = \dfrac{\cos x}{\sin x}$

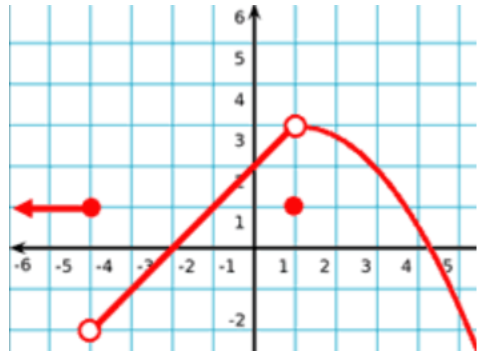
Evaluate the following using the definitions above and the unit circle!

a) $\tan\left(\frac{\pi}{6}\right)$ b) $\tan(0)$ c) $\tan\left(\frac{\pi}{4}\right)$ d) $\tan\left(\frac{\pi}{3}\right)$

e) $\tan\left(\frac{\pi}{2}\right)$ f) $\sec\left(\frac{\pi}{6}\right)$ g) $\sec(0)$ h) $\sec\left(\frac{\pi}{4}\right)$

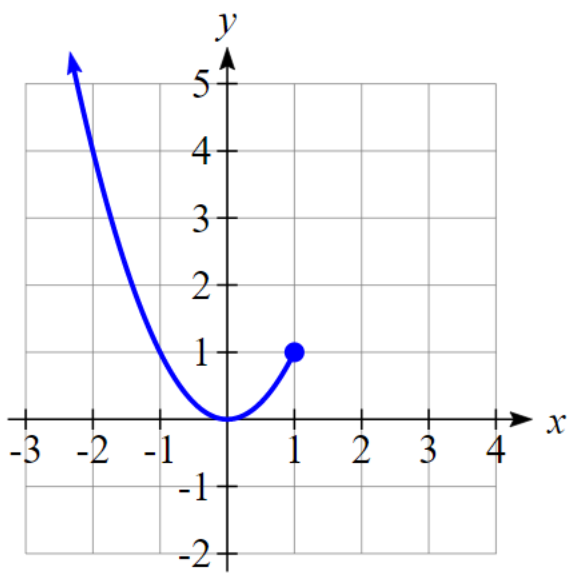
i) $\sec\left(\frac{\pi}{3}\right)$ j) $\sec\left(\frac{\pi}{2}\right)$ k) $\csc(0)$ l) $\csc\left(\frac{\pi}{4}\right)$

m) $\csc\left(\frac{\pi}{3}\right)$ n) $\cot\left(\frac{\pi}{2}\right)$ o) $\cot\left(\frac{\pi}{6}\right)$ p) $\cot(0)$



Given the graph of the function $f(x)$ above answer the following parts!

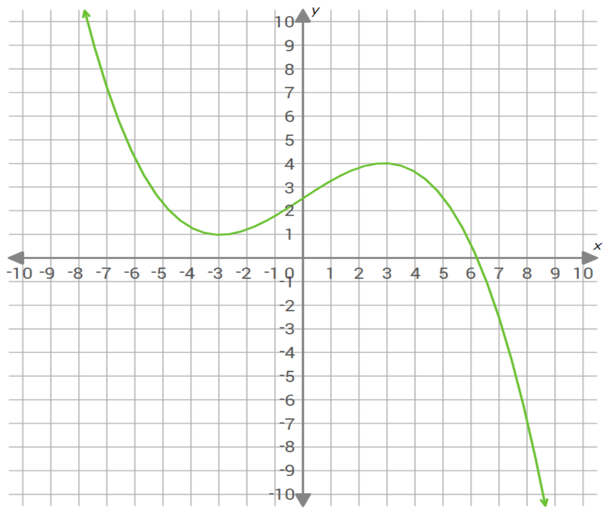
a) $f(-4) =$ b) $f(-3) =$ c) $f(-2) =$ d) $f(1) =$



Given the graph above answer the following parts! Make sure you use interval notation.

a) Domain: b) Range:

c) x-intercept(s): d) y-intercept:



Given the graph above answer the following parts! Make sure you use interval notation.

a) Domain:

b) Range:

c) x-intercept(s):

d) y-intercept:

e) Increasing:

f) Decreasing:

g) End behavior:

Congratulations! You have completed the AP Calculus summer assignment :)